Statistical Relational Learning

Pedro Domingos
Dept. of Computer Science & Eng.
University of Washington

Overview

- Motivation
- Foundational areas
  - Probabilistic inference
  - Statistical learning
  - Logical inference
  - Inductive logic programming
- Putting the pieces together
- Applications

Motivation

- Most learners assume i.i.d. data (independent and identically distributed)
  - One type of object
  - Objects have no relation to each other
- Real applications: dependent, variously distributed data
  - Multiple types of objects
  - Relations between objects

Examples

- Web search
- Information extraction
- Natural language processing
- Perception
- Medical diagnosis
- Computational biology
- Social networks
- Ubiquitous computing
- Etc.

Costs and Benefits of SRL

- **Benefits**
  - Better predictive accuracy
  - Better understanding of domains
  - Growth path for machine learning
- **Costs**
  - Learning is much harder
  - Inference becomes a crucial issue
  - Greater complexity for user

Goal and Progress

- **Goal:**
  Learn from non-i.i.d. data as easily as from i.i.d. data
- Progress to date
  - Burgeoning research area
  - We're "close enough" to goal
  - Easy-to-use open-source software available
- Lots of research questions (old and new)

Plan

- We have the elements:
  - **Probability** for handling uncertainty
  - **Logic** for representing types, relations, and complex dependencies between them
  - **Learning** and **inference** algorithms for each
- Figure out how to put them together
- Tremendous leverage on a wide range of applications

Disclaimers

- Not a complete survey of statistical relational learning
- Or of foundational areas
- Focus is practical, not theoretical
- Assumes basic background in logic, probability and statistics, etc.
- Please ask questions
- Tutorial and examples available at **alchemy.cs.washington.edu**

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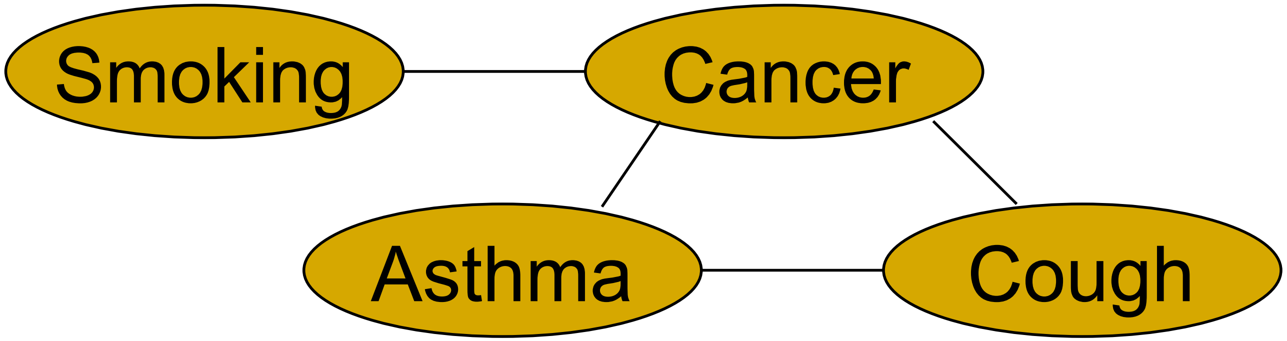
Markov Networks

- **Undirected** graphical models



- Potential functions defined over cliques

$$P(x) = \frac{1}{Z} \prod_c \Phi_c(x_c)$$

$$Z = \sum_x \prod_c \Phi_c(x_c)$$

Smoking	Cancer	Φ(S,C)
False	False	4.5
False	True	4.5
True	False	2.7
True	True	4.5

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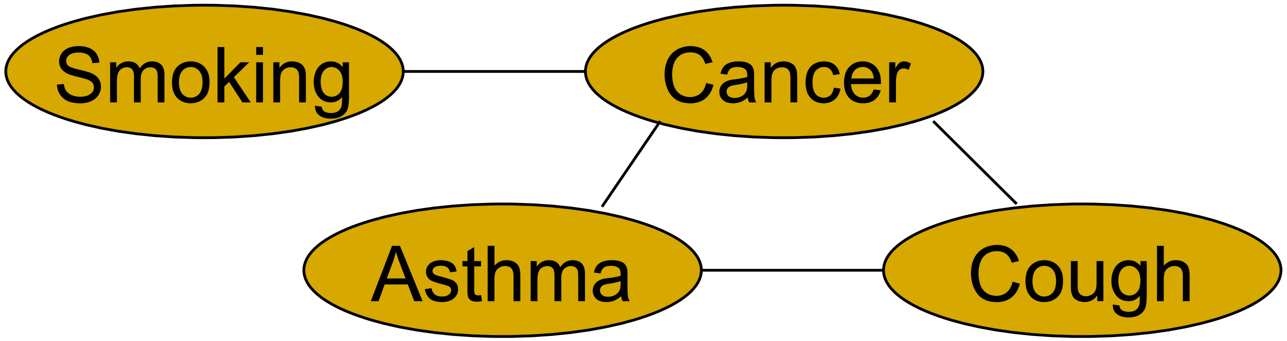
- **Undirected** graphical models



- Log-linear model:

$$P(x) = \frac{1}{Z} \exp\left(\sum_i \underbrace{w_i}_{\text{Weight of Feature } i} \underbrace{f_i(x)}_{\text{Feature } i} \right)$$

$$f_1(\text{Smoking, Cancer}) = \begin{cases} 1 & \text{if } \neg \text{ Smoking } \vee \text{ Cancer} \\ 0 & \text{otherwise} \end{cases}$$

$$w_1 = 1.5$$

Hammersley-Clifford Theorem

If Distribution is strictly positive (P(x) > 0)

And Graph encodes conditional independences

Then Distribution is product of potentials over cliques of graph

Inverse is also true.

("Markov network = Gibbs distribution")

Markov Nets vs. Bayes Nets

Property	Markov Nets	Bayes Nets
Form	Prod. potentials	Prod. potentials
Potentials	Arbitrary	Cond. probabilities
Cycles	Allowed	Forbidden
Partition func.	Z = ?	Z = 1
Indep. check	Graph separation	D-separation
Indep. props.	Some	Some
Inference	MCMC, BP, etc.	Convert to Markov

Inference in Markov Networks

- **Goal**: Compute marginals & conditionals of

$$P(X) = \frac{1}{Z} \exp\left(\sum_i w_i f_i(X) \right) \qquad Z = \sum_X \exp\left(\sum_i w_i f_i(X) \right)$$

- Exact inference is #P-complete
- Conditioning on Markov blanket is easy:

$$P(x \mid MB(x)) = \frac{\exp\left(\sum_i w_i f_i(x)\right)}{\exp\left(\sum_i w_i f_i(x=0)\right) + \exp\left(\sum_i w_i f_i(x=1)\right)}$$

- Gibbs sampling exploits this

MCMC: Gibbs Sampling

state ← random truth assignment
for *i* ← 1 **to** *num-samples* **do**
  for each variable *x*
    sample *x* according to P(*x*|*neighbors*(*x*))
    state ← *state* with new value of *x*
P(*F*) ← fraction of states in which *F* is true

Other Inference Methods

- Many variations of MCMC
- Belief propagation (sum-product)
- Variational approximation
- Exact methods

MAP/MPE Inference

- **Goal:** Find most likely state of world given evidence

$$\max_{y} P(y \mid x)$$

Query

Evidence

MAP Inference Algorithms

- Iterated conditional modes
- Simulated annealing
- Graph cuts
- Belief propagation (max-product)

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Learning Markov Networks

- Learning parameters (weights)
  - Generatively
  - Discriminatively
- Learning structure (features)
- In this tutorial: Assume complete data
  (If not: EM versions of algorithms)

Generative Weight Learning

- Maximize likelihood or posterior probability
- Numerical optimization (gradient or 2nd order)
- No local maxima

$$\frac{\partial}{\partial w_i} \log P_w(x) = \boxed{n_i(x)} - \boxed{E_w\left[n_i(x)\right]}$$

No. of times feature *i* is true in data

Expected no. times feature *i* is true according to model

- Requires inference at each step (slow!)

Pseudo-Likelihood

$$PL(x) \equiv \prod_i P(x_i \mid neighbors(x_i))$$

- Likelihood of each variable given its neighbors in the data
- Does not require inference at each step
- Consistent estimator
- Widely used in vision, spatial statistics, etc.
- But PL parameters may not work well for long inference chains

Discriminative Weight Learning

- Maximize conditional likelihood of query (y) given evidence (x)

$$\frac{\partial}{\partial w_i} \log P_w(y \mid x) = \boxed{n_i(x, y)} - \boxed{E_w\left[n_i(x, y)\right]}$$

No. of true groundings of clause *i* in data

Expected no. true groundings according to model

- Approximate expected counts by counts in MAP state of y given x

Other Weight Learning Approaches

- **Generative:** Iterative scaling
- **Discriminative:** Max margin

Structure Learning

- Start with atomic features
- Greedily conjoin features to improve score
- Problem: Need to reestimate weights for each new candidate
- Approximation: Keep weights of previous features constant

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First-Order Logic

- Constants, variables, functions, predicates
  E.g.: Anna, x, MotherOf(x), Friends(x, y)
- **Literal:** Predicate or its negation
- **Clause:** Disjunction of literals
- **Grounding:** Replace all variables by constants
  E.g.: Friends (Anna, Bob)
- **World** (model, interpretation):
  Assignment of truth values to all ground predicates

Inference in First-Order Logic

- Traditionally done by theorem proving (e.g.: Prolog)
- Propositionalization followed by model checking turns out to be faster (often a lot)
- **Propositionalization:** Create all ground atoms and clauses
- **Model checking:** Satisfiability testing
- Two main approaches:
  - **Backtracking** (e.g.: DPLL)
  - **Stochastic local search** (e.g.: WalkSAT)

Satisfiability

- **Input:** Set of clauses
  (Convert KB to conjunctive normal form (CNF))
- **Output:** Truth assignment that satisfies all clauses, or failure
- The paradigmatic NP-complete problem
- **Solution:** Search
- **Key point:**
  Most SAT problems are actually easy
- **Hard region:** Narrow range of
  #Clauses / #Variables

Backtracking

- Assign truth values by depth-first search
- Assigning a variable deletes false literals and satisfied clauses
- Empty set of clauses: Success
- Empty clause: Failure
- Additional improvements:
  - **Unit propagation** (unit clause forces truth value)
  - **Pure literals** (same truth value everywhere)

The DPLL Algorithm

if *CNF* is empty **then**
    return *true*
else if *CNF* contains an empty clause **then**
    return *false*
else if *CNF* contains a pure literal *x* **then**
    return *DPLL(CNF(x))*
else if *CNF* contains a unit clause {*u*} then
    return *DPLL(CNF(u))*
else
    choose a variable *x* that appears in *CNF*
    if *DPLL(CNF(x))* = *true* **then return** *true*
    else return *DPLL(CNF(¬x))*

Stochastic Local Search

- Uses complete assignments instead of partial
- Start with random state
- Flip variables in unsatisfied clauses
- Hill-climbing: Minimize # unsatisfied clauses
- Avoid local minima: Random flips
- Multiple restarts

The WalkSAT Algorithm

```
for i ← 1 to max-tries do
    solution = random truth assignment
    for j ← 1 to max-flips do
        if all clauses satisfied then
            return solution
        c ← random unsatisfied clause
        with probability p
            flip a random variable in c
        else
            flip variable in c that maximizes
                number of satisfied clauses
return failure
```

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Rule Induction

- **Given:** Set of positive and negative examples of some concept
  - **Example:** $(x_1, x_2, \ldots, x_n, y)$
  - y: **concept** (Boolean)
  - $x_1, x_2, \ldots, x_n$: **attributes** (assume Boolean)
- **Goal:** Induce a set of rules that cover all positive examples and no negative ones
  - **Rule:** $x_a \wedge x_b \wedge \ldots \Rightarrow y$ (x_a: Literal, i.e., x_i or its negation)
  - Same as **Horn clause**: *Body* $\Rightarrow$ *Head*
  - Rule *r* **covers** example *x* iff *x* satisfies body of *r*
- ***Eval(r):*** Accuracy, info. gain, coverage, support, etc.

Learning a Single Rule

head $\leftarrow$ *y*
body $\leftarrow$ Ø
repeat
    for each literal *x*
        $r_x \leftarrow r$ with *x* added to *body*
        $Eval(r_x)$
    body $\leftarrow$ *body* ^ best *x*
until no *x* improves *Eval(r)*
return *r*

Learning a Set of Rules

$R \leftarrow \emptyset$
$S \leftarrow examples$
repeat
    learn a single rule r
    $R \leftarrow R \cup \{ r \}$
    $S \leftarrow$ S − positive examples covered by r
until S contains no positive examples
return R

First-Order Rule Induction

- *y* and x_i are now predicates with arguments
  E.g.: *y* is Ancestor(x,y), x_i is Parent(x,y)
- Literals to add are predicates or their negations
- Literal to add must include at least one variable already appearing in rule
- Adding a literal changes # groundings of rule
  E.g.: Ancestor(x,z) ^ Parent(z,y) ⇒ Ancestor(x,y)
- *Eval(r)* must take this into account
  E.g.: Multiply by # positive groundings of rule
  still covered after adding literal

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Plethora of Approaches

- Knowledge-based model construction [Wellman et al., 1992]
- Stochastic logic programs [Muggleton, 1996]
- Probabilistic relational models [Friedman et al., 1999]
- Relational Markov networks [Taskar et al., 2002]
- Bayesian logic [Milch et al., 2005]
- Markov logic [Richardson & Domingos, 2006]
- And many others!

Key Dimensions

- **Logical language**
  First-order logic, Horn clauses, frame systems
- **Probabilistic language**
  Bayes nets, Markov nets, PCFGs
- **Type of learning**
  - Generative / Discriminative
  - Structure / Parameters
  - Knowledge-rich / Knowledge-poor
- **Type of inference**
  - MAP / Marginal
  - Full grounding / Partial grounding / Lifted

Knowledge-Based Model Construction

- **Logical language:** Horn clauses
- **Probabilistic language:** Bayes nets
  - Ground atom → Node
  - Head of clause → Child node
  - Body of clause → Parent nodes
  - >1 clause w/ same head → Combining function
- **Learning:** ILP + EM
- **Inference:** Partial grounding + Belief prop.

Stochastic Logic Programs

- **Logical language:** Horn clauses
- **Probabilistic language:** Probabilistic context-free grammars
  - Attach probabilities to clauses
  - .Σ Probs. of clauses w/ same head = 1
- **Learning:** ILP + “Failure-adjusted” EM
- **Inference:** Do all proofs, add probs.

Probabilistic Relational Models

- **Logical language:** Frame systems
- **Probabilistic language:** Bayes nets
  - Bayes net template for each class of objects
  - Object's attrs. can depend on attrs. of related objs.
  - Only binary relations
  - No dependencies of relations on relations
- **Learning:**
  - Parameters: Closed form (EM if missing data)
  - Structure: "Tiered" Bayes net structure search
- **Inference:** Full grounding + Belief propagation

Relational Markov Networks

- **Logical language:** SQL queries
- **Probabilistic language:** Markov nets
  - SQL queries define cliques
  - Potential function for each query
  - No uncertainty over relations
- **Learning:**
  - Discriminative weight learning
  - No structure learning
- **Inference:** Full grounding + Belief prop.

Bayesian Logic

- **Logical language:** First-order semantics
- **Probabilistic language:** Bayes nets
  - BLOG program specifies how to generate relational world
  - Parameters defined separately in Java functions
  - Allows unknown objects
  - May create Bayes nets with directed cycles
- **Learning:** None to date
- **Inference:**
  - MCMC with user-supplied proposal distribution
  - Partial grounding

Markov Logic

- **Logical language:** First-order logic
- **Probabilistic language:** Markov networks
  - **Syntax:** First-order formulas with weights
  - **Semantics:** Templates for Markov net features
- **Learning:**
  - **Parameters:** Generative or discriminative
  - **Structure:** ILP with arbitrary clauses and MAP score
- **Inference:**
  - **MAP:** Weighted satisfiability
  - **Marginal:** MCMC with moves proposed by SAT solver
  - Partial grounding + Lazy inference

Markov Logic

- Most developed approach to date
- Many other approaches can be viewed as special cases
- Main focus of rest of this tutorial

Markov Logic: Intuition

- A logical KB is a set of **hard constraints** on the set of possible worlds
- Let's make them **soft constraints**:
When a world violates a formula,
It becomes less probable, not impossible
- Give each formula a **weight**
(Higher weight $\Rightarrow$ Stronger constraint)

$$P(\text{world}) \propto \exp\left(\sum \text{weights of formulas it satisfies}\right)$$

Markov Logic: Definition

- A Markov Logic Network (MLN) is a set of pairs (F, w) where
  - F is a formula in first-order logic
  - w is a real number
- Together with a set of constants, it defines a Markov network with
  - One node for each grounding of each predicate in the MLN
  - One feature for each grounding of each formula F in the MLN, with the corresponding weight w

Example: Friends & Smokers

Smoking causes cancer.

Friends have similar smoking habits.

Example: Friends & Smokers

$$\forall x \; Smokes(x) \Rightarrow Cancer(x)$$
$$\forall x, y \; Friends(x, y) \Rightarrow \left(Smokes(x) \Leftrightarrow Smokes(y)\right)$$

Example: Friends & Smokers

1.5	$\forall x\ Smokes(x) \Rightarrow Cancer(x)$
1.1	$\forall x, y\ Friends(x, y) \Rightarrow \left(Smokes(x) \Leftrightarrow Smokes(y)\right)$

Example: Friends & Smokers

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Two constants: **Anna** (A) and **Bob** (B)

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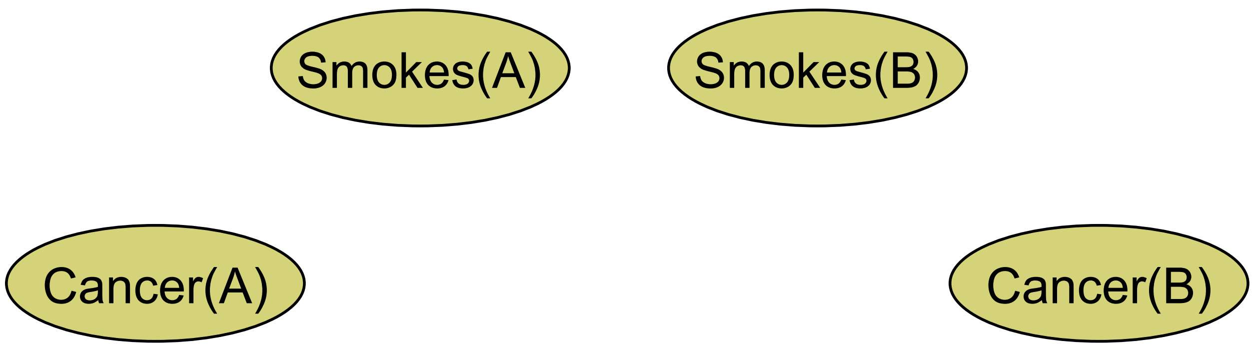

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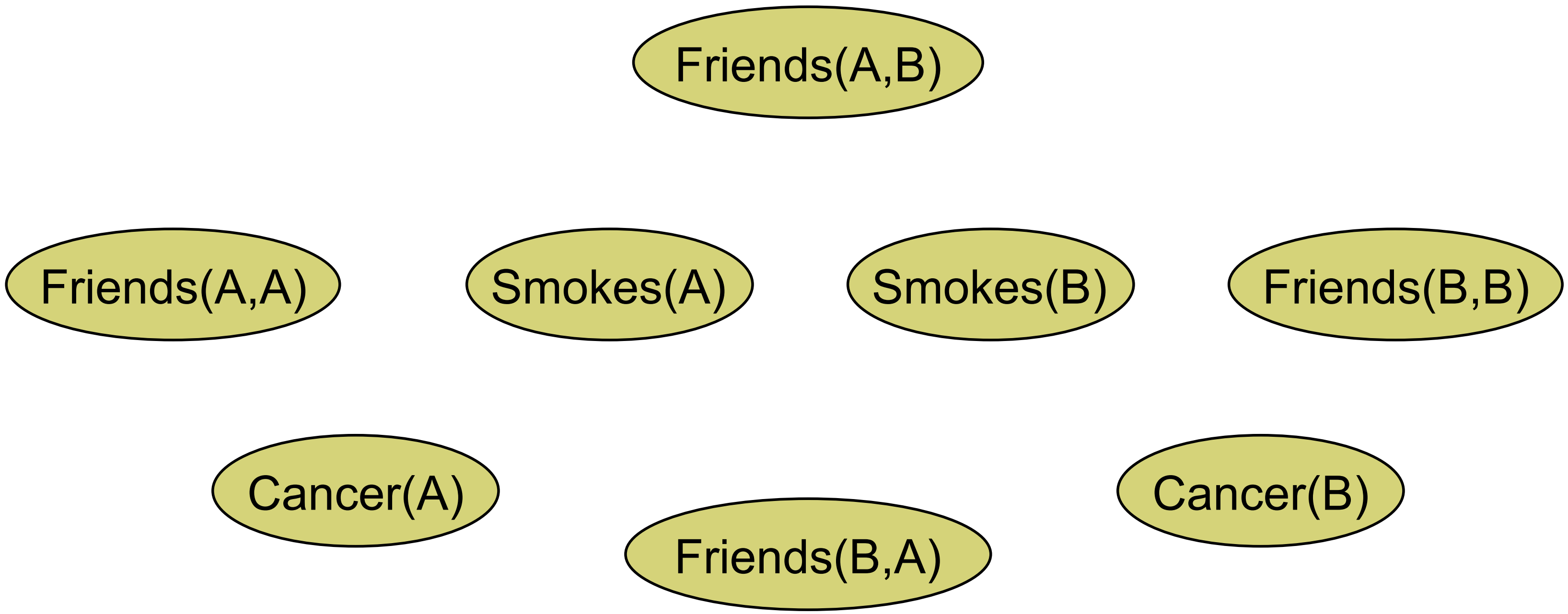

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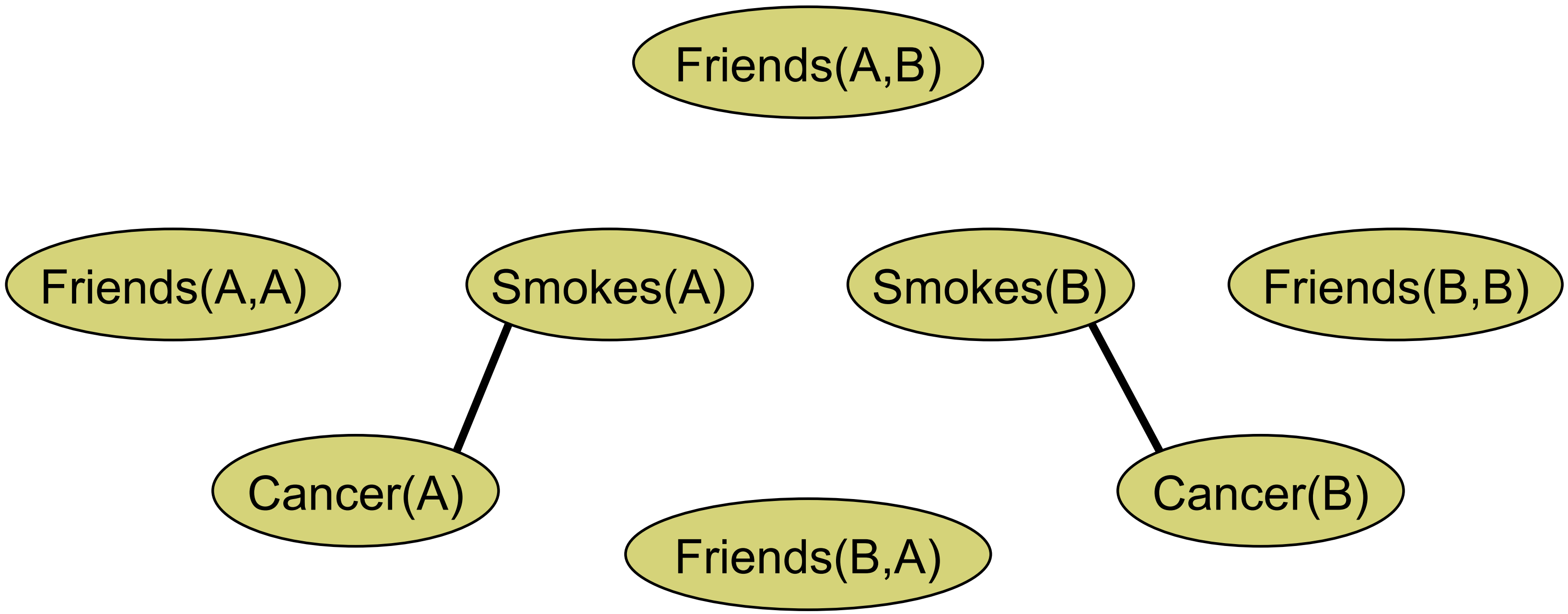

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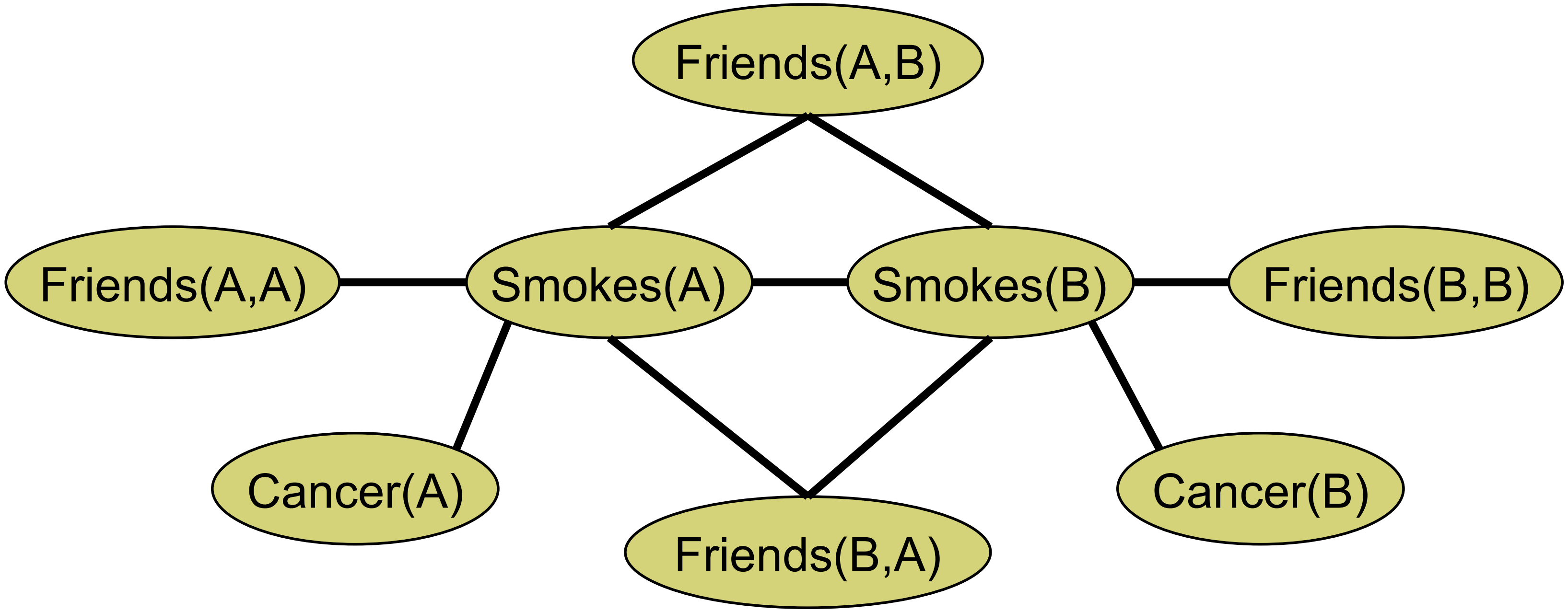

Markov Logic Networks

- MLN is **template** for ground Markov nets
- Probability of a world x:

$$P(x) = \frac{1}{Z} \exp\left(\sum_i \underbrace{w_i}_{\text{Weight of formula } i} \underbrace{n_i(x)}_{\text{No. of true groundings of formula } i \text{ in } x} \right)$$

- **Typed** variables and constants greatly reduce size of ground Markov net
- Functions, existential quantifiers, etc.
- Infinite and continuous domains

Relation to Statistical Models

- Special cases:
  - Markov networks
  - Markov random fields
  - Bayesian networks
  - Log-linear models
  - Exponential models
  - Max. entropy models
  - Gibbs distributions
  - Boltzmann machines
  - Logistic regression
  - Hidden Markov models
  - Conditional random fields
- Obtained by making all predicates zero-arity
- Markov logic allows objects to be interdependent (non-i.i.d.)

Relation to First-Order Logic

- Infinite weights $\Rightarrow$ First-order logic
- Satisfiable KB, positive weights $\Rightarrow$ Satisfying assignments = Modes of distribution
- Markov logic allows contradictions between formulas

MAP/MPE Inference

- **Problem:** Find most likely state of world given evidence

$$\underset{y}{\arg\max}\; P(\underbrace{y}_{\text{Query}} \mid \underbrace{x}_{\text{Evidence}})$$

MAP/MPE Inference

- **Problem:** Find most likely state of world given evidence

$$\arg\max_{y} \frac{1}{Z_x} \exp\left(\sum_i w_i n_i(x, y) \right)$$

MAP/MPE Inference

- **Problem:** Find most likely state of world given evidence

$$\arg\max_{y} \sum_{i} w_i n_i(x, y)$$

MAP/MPE Inference

- **Problem:** Find most likely state of world given evidence

$$\arg\max_{y} \sum_{i} w_i n_i(x, y)$$

- This is just the weighted MaxSAT problem
- Use weighted SAT solver (e.g., MaxWalkSAT [Kautz et al., 1997])
- Potentially faster than logical inference (!)

The MaxWalkSAT Algorithm

for *i* ← 1 to *max-tries* **do**
    solution = random truth assignment
    for *j* ← 1 **to** *max-flips* **do**
        if ∑ weights(sat. clauses) > threshold **then**
            return *solution*
        c ← random unsatisfied clause
        with probability *p*
            flip a random variable in *c*
        else
            flip variable in *c* that maximizes
                ∑ weights(sat. clauses)
return failure, best *solution* found

But … Memory Explosion

- **Problem:**
  If there are **n** constants
  and the highest clause arity is **c**,
  the ground network requires $\mathbf{O(n^c)}$ memory

- **Solution:**
  Exploit sparseness; ground clauses lazily
  → LazySAT algorithm [Singla & Domingos, 2006]

Computing Probabilities

- P(Formula|MLN,C) = ?
- MCMC: Sample worlds, check formula holds
- P(Formula1|Formula2,MLN,C) = ?
- If Formula2 = Conjunction of ground atoms
  - First construct min subset of network necessary to answer query (generalization of KBMC)
  - Then apply MCMC (or other)
- Can also do lifted inference [Braz et al, 2005]

Ground Network Construction

network ← Ø
queue ← query nodes
repeat
    node ← front(*queue*)
    remove *node* from *queue*
    add *node* to *network*
    if *node* not in evidence **then**
        add neighbors(*node*) to queue
until queue = Ø

But … Insufficient for Logic

- **Problem:**
  Deterministic dependencies break MCMC
  Near-deterministic ones make it ***very*** slow

- **Solution:**
  Combine MCMC and WalkSAT
  $\longrightarrow$ MC-SAT algorithm [Poon & Domingos, 2006]

Learning

- Data is a relational database
- Closed world assumption (if not: EM)
- Learning parameters (weights)
- Learning structure (formulas)

Weight Learning

- Parameter tying: Groundings of same clause

$$\frac{\partial}{\partial w_i} \log P_w(x) = \boxed{n_i(x)} - \boxed{E_w\left[n_i(x)\right]}$$

No. of times clause *i* is true in data

Expected no. times clause *i* is true according to MLN

- Generative learning: Pseudo-likelihood
- Discriminative learning: Cond. likelihood, use MC-SAT or MaxWalkSAT for inference

Structure Learning

- Generalizes feature induction in Markov nets
- Any inductive logic programming approach can be used, but . . .
- Goal is to induce any clauses, not just Horn
- Evaluation function should be likelihood
- Requires learning weights for each candidate
- Turns out not to be bottleneck
- Bottleneck is counting clause groundings
- Solution: Subsampling

Structure Learning

- **Initial state:** Unit clauses or hand-coded KB
- **Operators:** Add/remove literal, flip sign
- **Evaluation function:**
  Pseudo-likelihood + Structure prior
- **Search:** Beam, shortest-first, bottom-up
  [Kok & Domingos, 2005; Mihalkova & Mooney, 2007]

Alchemy

Open-source software including:

- Full first-order logic syntax
- Generative & discriminative weight learning
- Structure learning
- Weighted satisfiability and MCMC
- Programming language features

alchemy.cs.washington.edu

	Alchemy	**Prolog**	**BUGS**
Represent-ation	F.O. Logic + Markov nets	Horn clauses	Bayes nets
Inference	Model check-ing, MC-SAT	Theorem proving	Gibbs sampling
Learning	Parameters & structure	No	Params.
Uncertainty	Yes	No	Yes
Relational	Yes	Yes	No

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Applications

- Basics
- Logistic regression
- Hypertext classification
- Information retrieval
- Entity resolution
- Hidden Markov models
- Information extraction
- Statistical parsing
- Semantic processing
- Bayesian networks
- Relational models
- Robot mapping
- Planning and MDPs
- Practical tips

Running Alchemy

- Programs
  - Infer
  - Learnwts
  - Learnstruct
- Options
- MLN file
  - Types (optional)
  - Predicates
  - Formulas
- Database files

Uniform Distribn.: Empty MLN

Example: Unbiased coin flips

Type: `flip = { 1, … , 20 }`

Predicate: `Heads(flip)`

$$P(Heads(f)) = \frac{\frac{1}{Z}e^0}{\frac{1}{Z}e^0 + \frac{1}{Z}e^0} = \frac{1}{2}$$

Binomial Distribn.: Unit Clause

Example: Biased coin flips

Type: `flip = { 1, … , 20 }`

Predicate: `Heads(flip)`

Formula: `Heads(f)`

Weight: Log odds of heads: $w = \log\left(\frac{p}{1-p}\right)$

$$P(\text{Heads(f)}) = \frac{\frac{1}{Z}e^{w}}{\frac{1}{Z}e^{w} + \frac{1}{Z}e^{0}} = \frac{1}{1+e^{-w}} = p$$

By default, MLN includes unit clauses for all predicates (captures marginal distributions, etc.)

Multinomial Distribution

Example: Throwing die

```
Types:      throw = { 1, … , 20 }
            face = { 1, … , 6 }
Predicate:  Outcome(throw,face)
Formulas:   Outcome(t,f) ^ f != f' => !Outcome(t,f').
            Exist f Outcome(t,f).
```

Too cumbersome!

Multinomial Distrib.: ! Notation

Example: Throwing die

Types: `throw = { 1, … , 20 }`

`face = { 1, … , 6 }`

Predicate: `Outcome(throw,face!)`

Formulas:

Semantics: Arguments without “!” determine arguments with “!”.

Also makes inference more efficient (triggers blocking).

Multinomial Distrib.: + Notation

Example: Throwing biased die

Types: `throw = { 1, … , 20 }`

`face = { 1, … , 6 }`

Predicate: `Outcome(throw,face!)`

Formulas: `Outcome(t,+f)`

Semantics: Learn weight for each grounding of args with “+”.

Logistic Regression

Logistic regression: $\log\left(\frac{P(C=1\,|\,\mathbf{F}=\mathbf{f})}{P(C=0\,|\,\mathbf{F}=\mathbf{f})}\right) = a + \sum b_i f_i$

Type: `obj = { 1, ... , n }`
Query predicate: `C(obj)`
Evidence predicates: F_i`(obj)`
Formulas: `a  C(x)`
b_i F_i`(x) ^ C(x)`

Resulting distribution: $P(C=c,\mathbf{F}=\mathbf{f}) = \frac{1}{Z}\exp\left(ac + \sum_i b_i f_i c\right)$

Therefore: $\log\left(\frac{P(C=1\,|\,\mathbf{F}=\mathbf{f})}{P(C=0\,|\,\mathbf{F}=\mathbf{f})}\right) = \log\left(\frac{\exp\left(a + \sum b_i f_i\right)}{\exp(0)}\right) = a + \sum b_i f_i$

Alternative form: F_i`(x) => C(x)`

Text Classification

```
page = { 1, … , n }
word = { … }
topic = { … }

Topic(page,topic!)
HasWord(page,word)

!Topic(p,t)
HasWord(p,+w) => Topic(p,+t)
```

Text Classification

```
Topic(page,topic!)
HasWord(page,word)

HasWord(p,+w) => Topic(p,+t)
```

Hypertext Classification

```
Topic(page,topic!)
HasWord(page,word)
Links(page,page)

HasWord(p,+w) => Topic(p,+t)
Topic(p,t) ^ Links(p,p') => Topic(p',t)
```

Cf. S. Chakrabarti, B. Dom & P. Indyk, "Hypertext Classification Using Hyperlinks," in *Proc. SIGMOD-1998*.

Information Retrieval

```
InQuery(word)
HasWord(page,word)
Relevant(page)

InQuery(w+) ^ HasWord(p,+w) => Relevant(p)
Relevant(p) ^ Links(p,p') => Relevant(p')
```

Cf. L. Page, S. Brin, R. Motwani & T. Winograd, “The PageRank Citation Ranking: Bringing Order to the Web,” Tech. Rept., Stanford University, 1998.

Entity Resolution

Problem: Given database, find duplicate records

```
HasToken(token,field,record)
SameField(field,record,record)
SameRecord(record,record)

HasToken(+t,+f,r) ^ HasToken(+t,+f,r′)
   => SameField(f,r,r′)
SameField(f,r,r′) => SameRecord(r,r′)
SameRecord(r,r′) ^ SameRecord(r′,r″)
   => SameRecord(r,r″)
```

Cf. A. McCallum & B. Wellner, "Conditional Models of Identity Uncertainty with Application to Noun Coreference," in *Adv. NIPS 17*, 2005.

Entity Resolution

Can also resolve fields:

```
HasToken(token,field,record)
SameField(field,record,record)
SameRecord(record,record)

HasToken(+t,+f,r) ^ HasToken(+t,+f,r')
   => SameField(f,r,r')
SameField(f,r,r') <=> SameRecord(r,r')
SameRecord(r,r') ^ SameRecord(r',r")
   => SameRecord(r,r")
SameField(f,r,r') ^ SameField(f,r',r")
   => SameField(f,r,r")
```

More: P. Singla & P. Domingos, "Entity Resolution with Markov Logic", in *Proc. ICDM-2006*.

Hidden Markov Models

```
obs = { Obs1, … , ObsN }
state = { St1, … , StM }
time = { 0, … , T }

State(state!,time)
Obs(obs!,time)

State(+s,0)
State(+s,t) => State(+s',t+1)
Obs(+o,t) => State(+s,t)
```

Information Extraction

- **Problem:** Extract database from text or semi-structured sources
- **Example:** Extract database of publications from citation list(s) (the “CiteSeer problem”)
- Two steps:
  - **Segmentation:**
    Use HMM to assign tokens to fields
  - **Entity resolution:**
    Use logistic regression and transitivity

Information Extraction

```
Token(token, position, citation)
InField(position, field, citation)
SameField(field, citation, citation)
SameCit(citation, citation)

Token(+t,i,c) => InField(i,+f,c)
InField(i,+f,c) <=> InField(i+1,+f,c)
f != f′ => (!InField(i,+f,c) v !InField(i,+f′,c))

Token(+t,i,c) ^ InField(i,+f,c) ^ Token(+t,i′,c′)
   ^ InField(i′,+f,c′) => SameField(+f,c,c′)
SameField(+f,c,c′) <=> SameCit(c,c′)
SameField(f,c,c′) ^ SameField(f,c′,c″) => SameField(f,c,c″)
SameCit(c,c′) ^ SameCit(c′,c″) => SameCit(c,c″)
```

Information Extraction

```
Token(token, position, citation)
InField(position, field, citation)
SameField(field, citation, citation)
SameCit(citation, citation)

Token(+t,i,c) => InField(i,+f,c)
InField(i,+f,c) ^ !Token(".",i,c) <=> InField(i+1,+f,c)
f != f' => (!InField(i,+f,c) v !InField(i,+f',c))

Token(+t,i,c) ^ InField(i,+f,c) ^ Token(+t,i',c')
   ^ InField(i',+f,c') => SameField(+f,c,c')
SameField(+f,c,c') <=> SameCit(c,c')
SameField(f,c,c') ^ SameField(f,c',c") => SameField(f,c,c")
SameCit(c,c') ^ SameCit(c',c") => SameCit(c,c")
```

More: H. Poon & P. Domingos, "Joint Inference in Information Extraction", in *Proc. AAAI-2007*.

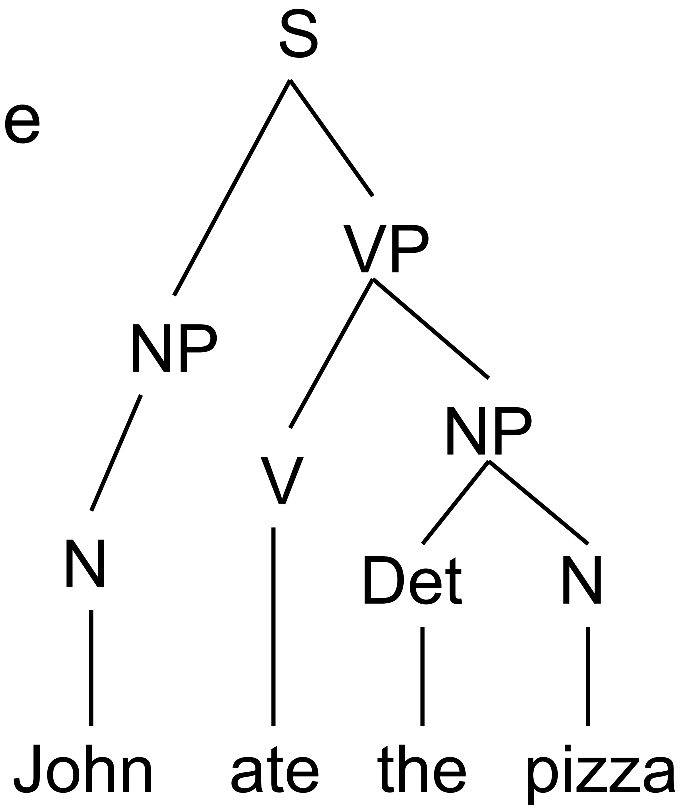
Statistical Parsing

- **Input:** Sentence
- **Output:** Most probable parse
- **PCFG:** Production rules with probabilities

  E.g.: 0.7 NP → N

  0.3 NP → Det N
- **WCFG:** Production rules with weights (equivalent)
- Chomsky normal form:

  A → B C or A → a



Statistical Parsing

- **Evidence predicate:** **`Token(token,position)`**
  E.g.: `Token("pizza", 3)`
- **Query predicates:** ***`Constituent`*** **`(position,position)`**
  E.g.: `NP(2,4)`
- For each rule of the form A → B C:
  Clause of the form **`B(i,j) ^ C(j,k) => A(i,k)`**
  E.g.: `NP(i,j) ^ VP(j,k) => S(i,k)`
- For each rule of the form A → a:
  Clause of the form **`Token(a,i) => A(i,i+1)`**
  E.g.: `Token("pizza", i) => N(i,i+1)`
- For each nonterminal:
  Hard formula stating that exactly one production holds
- MAP inference yields most probable parse

Semantic Processing

- **Weighted definite clause grammars:**
  Straightforward extension
- **Combine with entity resolution:**
  `NP(i,j) => Entity(+e,i,j)`
- **Word sense disambiguation:**
  Use logistic regression
- **Semantic role labeling:**
  Use rules involving phrase predicates
- **Building meaning representation:**
  Via weighted DCG with lambda calculus
  (cf. Zettlemoyer & Collins, UAI-2005)
- **Another option:**
  Rules of the form **`Token(a,i)`** **`=>`** ***`Meaning`***
  and ***`MeaningB`*** **`^`** ***`MeaningC`*** **`^`** … **`=>`** ***`MeaningA`***
- Facilitates injecting world knowledge into parsing

Semantic Processing

Example: John ate pizza.

Grammar: S → NP VP VP → V NP V → ate
NP → John NP → pizza

```
Token("John",0) => Participant(John,E,0,1)
Token("ate",1) => Event(Eating,E,1,2)
Token("pizza",2) => Participant(pizza,E,2,3)
Event(Eating,e,i,j) ^ Participant(p,e,j,k)
  ^ VP(i,k) ^ V(i,j) ^ NP(j,k) => Eaten(p,e)
Event(Eating,e,j,k) ^ Participant(p,e,i,j)
  ^ S(i,k) ^ NP(i,j) ^ VP(j,k) => Eater(p,e)
Event(t,e,i,k) => Isa(e,t)
```

Result: `Isa(E,Eating), Eater(John,E), Eaten(pizza,E)`

Bayesian Networks

- Use all binary predicates with same first argument (the object *x*).
- One predicate for each variable *A*: **`A(x,v!)`**
- One clause for each line in the CPT and value of the variable
- Context-specific independence:
  One Horn clause for each path in the decision tree
- Logistic regression: As before
- Noisy OR: Deterministic OR + Pairwise clauses

Relational Models

- **Knowledge-based model construction**
  - Allow only Horn clauses
  - Same as Bayes nets, except arbitrary relations
  - Combin. function: Logistic regression, noisy-OR or external
- **Stochastic logic programs**
  - Allow only Horn clauses
  - Weight of clause = log(p)
  - Add formulas: Head holds => Exactly one body holds
- **Probabilistic relational models**
  - Allow only binary relations
  - Same as Bayes nets, except first argument can vary

Relational Models

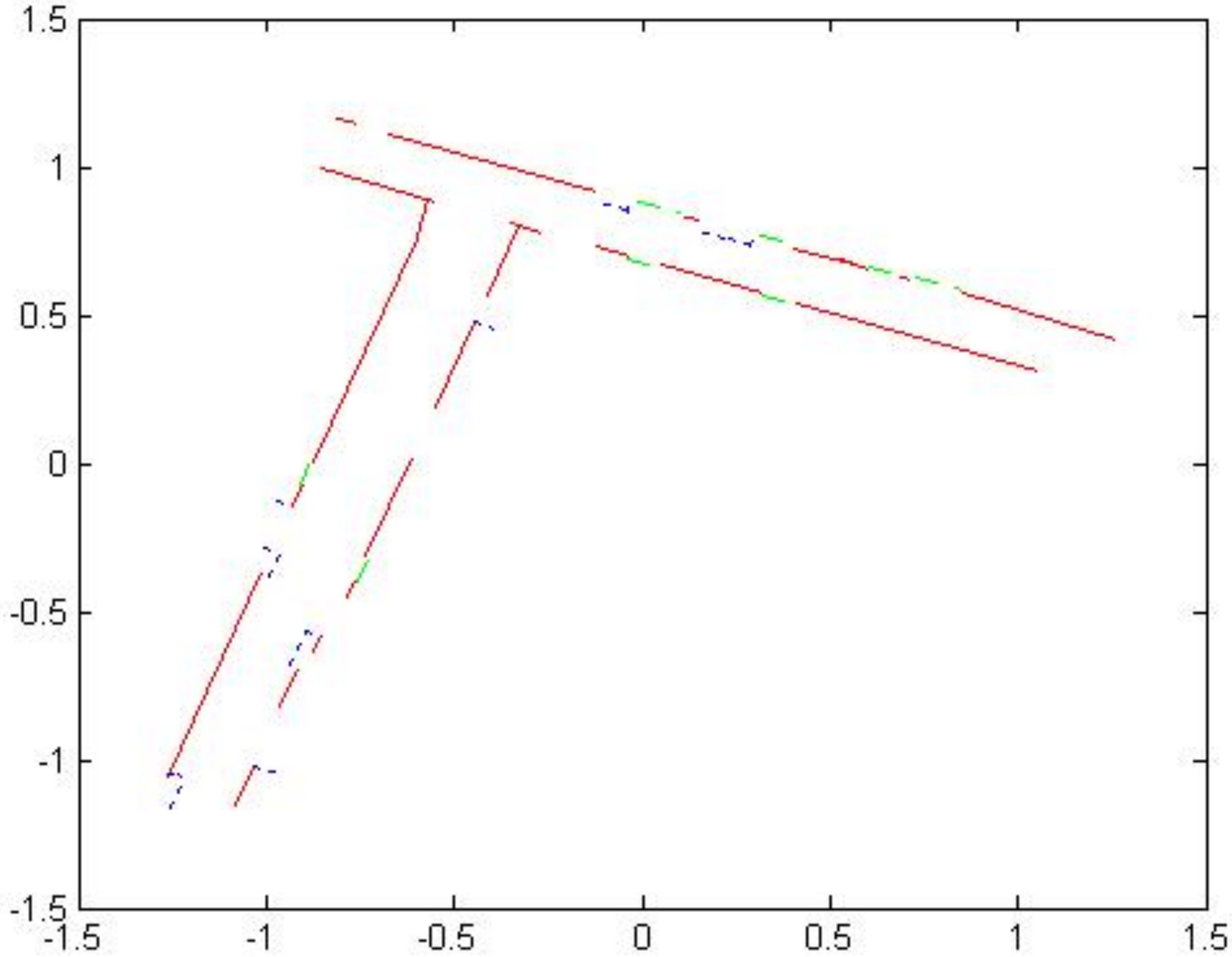
- **Relational Markov networks**
  - SQL → Datalog → First-order logic
  - One clause for each state of a clique
  - `*` syntax in Alchemy facilitates this
- **Bayesian logic**
  - Object = Cluster of similar/related observations
  - Observation constants + Object constants
  - Predicate `InstanceOf(Obs,Obj)` and clauses using it
- **Unknown relations:** Second-order Markov logic

  S. Kok & P. Domingos, "Statistical Predicate Invention", in *Proc. ICML-2007*. (Tomorrow at 3:15pm in Austin Auditorium)

Robot Mapping

- **Input:**
  Laser range finder segments (x_i, y_i, x_f, y_f)
- **Outputs:**
  - Segment labels (Wall, Door, Other)
  - Assignment of wall segments to walls
  - Position of walls (x_i, y_i, x_f, y_f)

Robot Mapping



MLNs for Hybrid Domains

- Allow numeric properties of objects as nodes

  E.g.: `Length(x)`, `Distance(x,y)`
- Allow numeric terms as features

  E.g.: $-(\texttt{Length(x)} - 5.0)^2$

  (Gaussian distr. w/ mean = 5.0 and variance = 1/(2w))
- Allow $\alpha = \beta$ as shorthand for $-(\alpha - \beta)^2$

  E.g.: `Length(x) = 5.0`
- Etc.

Robot Mapping

```
SegmentType(s,+t) => Length(s) = Length(+t)
SegmentType(s,+t) => Depth(s) = Depth(+t)
Neighbors(s,s′) ^ Aligned(s,s′) =>
   (SegType(s,+t) <=> SegType(s′,+t))
!PreviousAligned(s) ^ PartOf(s,l) => StartLine(s,l)
StartLine(s,l) => Xi(s) = Xi(l) ^ Yi(s) = Yi(l)
```

$$\texttt{PartOf(s,l)} \Rightarrow \frac{\texttt{Yf(s)-Yi(s)}}{\texttt{Xf(s)-Xi(s)}} = \frac{\texttt{Yi(s)-Yi(l)}}{\texttt{Xi(s)-Xi(l)}}$$

Etc.

Cf. B. Limketkai, L. Liao & D. Fox, “Relational Object Maps for Mobile Robots”, in *Proc. IJCAI-2005*.

Planning and MDPs

- **Classical planning**
  Formulate as satisfiability in the usual way
- **Actions with uncertain effects**
  Give finite weights to action axioms
- **Sensing actions**
  Add clauses relating sensor readings to world states
- **Relational Markov Decision Processes**
  - Assign utility weights to clauses (coming soon!)
  - Maximize expected sum of weights of satisfied utility clauses
  - Classical planning is special case:
    `Exist t GoalState(t)`

Practical Tips

- Add all unit clauses (the default)
- Implications vs. conjunctions
- Open/closed world assumptions
- How to handle uncertain data:
  `R(x,y) => R'(x,y)` (the "HMM trick")
- Controlling complexity
  - Low clause arities
  - Low numbers of constants
  - Short inference chains
- Use the simplest MLN that works
- Cycle: Add/delete formulas, learn and test

Summary

- Most domains are non-i.i.d.
- Much progress in recent years
- SRL mature enough to be practical tool
- Many old and new research issues
- Check out the Alchemy Web site: **alchemy.cs.washington.edu**